

Encontro Nacional da Sociedade Portuguesa de Matemática

Sessão temática Álgebra e Combinatória

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Organização: Teresa Maria Sousa, Escola Naval

	11 julho	12 julho
11h	Alan Cain	João Santos
11h30	António Malheiro	Paula Catarino
12h	Natália Rego	Paulo Guilherme dos Santos

Orador Alan Cain

Afiliação Centro de Matemática e Aplicações Faculdade de Ciências e Tecnologia - Universidade Nova de Lisboa

Título Combinatorics of cyclic shifts in the plactic, hypoplactic, sylvester, and related monoids

Resumo Young tableaux are the elements of the plactic monoid, and it was proved by Lascoux & Schutzenberger in their seminal study that if two of these elements contain the same number of each symbol, then one can be transformed to the other by applying a sequence of cyclic shifts (that is, a sequence of moves from an element that factors as xy to the element yx). Thus, if we build a "cyclic shift graph" whose vertices are elements of the monoid and whose edges connect elements that differ by a cyclic shift, then each connected component consists of precisely those elements that contain a given number of each generating symbol. Choffrut & Mercaş proved that in the plactic monoid of rank n (whose elements are Young tableaux whose entries are at most n), the number of cyclic shifts required is at most $2n-2$. That is, the diameter of connected components of the cyclic shift graph is at most $2n-2$ (although the number of elements they contain is unbounded). This talk discusses new results on the

cyclic shift graphs for a family of monoids that, like the plactic monoid, are closely connected with combinatorial objects: the hypoplactic monoid (connected with quasi-ribbon tableaux and quasi-symmetric functions), the sylvestermoid (binary search trees), the taiga monoid (binary search trees with multiplicities), and the stalactic monoid (stalactic tableaux). In each case, the diameter of connected components of the cyclic shift graph turns out to be dependent only on the rank of the monoid and not on the number of elements in a connected component. The proofs exploit the combinatorial objects associated to the monoids: similar monoids that have no such associated objects can have unbounded diameters of connected components. This is joint work with António Malheiro (Centro de Matemática e Aplicações & Departamento de Matemática, Universidade Nova de Lisboa).

Orador: António Malheiro

Afiliação: Departamento de Matemática e Centro de Matemática e Aplicações Faculdade de Ciências e Tecnologia -
Universidade Nova de Lisboa

Título: On the complexity of the word problem for the Plactic monoid.

Resumo: As argued by Schützenberger, the plactic monoid is “one of the most fundamental monoids in algebra”. It has its origins in work of Schensted and Knuth concerned with certain combinatorial problems and operations on Young tableaux. As a consequence of the Schensted insertion algorithm and the representation of elements by tableaux, it follows that the Plactic monoid has word problem that is solvable in quadratic time. This leads us naturally to the subject of automatic structures. The concept of an automatic group was introduced in order to describe a large class of groups with easily solvable word problem. The notion has been extended to automatic monoids and semigroups. In both cases the defining property is the existence of a rational set of normal forms (with respect to some finite generating set A) such that we have, for each generator in A , a finite automaton that recognizes pairs of normal forms that differ by multiplication by that generator. It is a consequence of the definition that automatic monoids (and in particular automatic groups) have word problem that is solvable in quadratic time. Our intention is to present the ideas above and to detail the case of automaticity for the Plactic monoid. This work was developed together with A. Cain and R. Gray and is based on the paper: Cain, Alan J., Robert D. Gray, and António Malheiro. "Finite Gröbner-Shirshov bases for plactic algebras and biautomatic structures for plactic monoids." *J. Algebra*. 423 (2015): 37-53

Orador: Natália Rego

Afiliação: IPCA, Departamento de Ciências - Barcelos, Portugal

Título: On universal central extensions of Hom-Leibniz algebras

Resumo: A Hom-Leibniz algebra \cite{MS} is a \mathbb{K} -vector space L endowed with a bilinear map $[-, -] : L \times L \rightarrow L$ and a \mathbb{K} -linear map $\alpha_L : L \rightarrow L$ satisfying, the so called Hom-Leibniz identity, $[\alpha_L(x), [y, z]] = [[x, y], \alpha_L(z)] - [[x, z], \alpha_L(y)]$, for all $x, y, z \in L$. If $[x, y] = -[y, x]$, then we have a Hom-Lie algebra. When the twisting map $\alpha_L = \text{Id}_L$, then the notion of Leibniz (Lie) algebra is recovered. The main goal of this talk is to present the generalization of classical results that characterize universal central extensions of Leibniz algebras to the framework of Hom-Leibniz algebras. Nevertheless, in this generalization fails the key result that claims the composition of central extensions is central as well. This singularity motivates the introduction of new concepts as α -perfect Hom-Leibniz algebra ($L = (\alpha_L(L), \alpha_L(L))$) and α -central extension ($0 \rightarrow (M, \alpha_M) \rightarrow (K, \alpha_K) \xrightarrow{\pi} (L, \alpha_L) \rightarrow 0$) is α -central if $\alpha_M(M) \subseteq Z(K)$. Then the corresponding characterizations are given. On the other hand, we analyze the relationship between the universal α -central extension of an α -perfect Hom-Lie algebra in the categories of Hom-Lie and Hom-Leibniz algebras (see \cite{CIP}).

palavras-chave: Hom-Leibniz algebra; homology; universal α -central extensions.

thebibliography{99}\bibitem{revista}J. M. Casas ;M. A. Insua; N. Pacheco Rego,, ``On Universal central extensions of Hom-Leibniz algebras'', \emph{Journal of Algebra and Its Applications}, Vol.13, No.8 (2014), pp. 1450053 (22 pages).\bibitem{CIP} J. M. Casas, M. A. Insua and N. Pacheco, ``On universal central extensions of Hom-Lie algebras'', \emph{Haceteppe Journal of Mathematics and Statistics}, Vol.44, No.2 (2015), pp. 277-288.\bibitem{MS} A. Makhlouf and S. Silvestrov, ``Hom-algebra structures'', \emph{J. Gen. Lie Theory Appl.} {\bf 2} (2) (2008),pp. 51--64.\end{thebibliography}\end{document}

Orador: João Santos

Afiliação: Department of Mathematics - University of Notre Dame, Indiana, USA

Título: Ideais binomiais de arestas

Resumo: Em 1975, Richard Stanley demonstrou a conjectura do majorante para esferas (uma conjectura de Combinatória) recorrendo à Álgebra Comutativa (mais precisamente, à teoria dos anéis Cohen-Macaulay). A partir daí nasceu um novo ramo da Matemática, a Álgebra Comutativa Combinatória, que, como o nome indica, consiste na intersecção da Álgebra Comutativa com a Combinatória. Em particular esta palestra consiste na apresentação de um tópico de interesse em Álgebra Comutativa introduzido em 2009: o estudo dos ideais binomiais de arestas. Fixado um corpo k , o ideal binomial de arestas do grafo G com vértices $\{1, \dots, n\}$ é o ideal de $k[x_1, \dots, x_n, y_1, \dots, y_n]$ gerado pelos binómios $x_i y_j - x_j y_i$, onde i e j são vértices adjacentes. As propriedades algébricas destes ideais (conjunto de primos minimais, dimensão, depth, condição de Cohen-Macaulay e regularidade) podem ser estudadas em função das propriedades combinatórias do grafo correspondente. Serão apresentados os resultados conhecidos sobre essas propriedades algébricas, bem como algumas conjecturas.

Oradora: Paula Catarino

Afiliação: Departamento de Matemática, Escola de Ciências e Tecnologia - Universidade de Trás-os-Montes e Alto Douro

Título: Some special sequences of quaternions and octonions

Resumo: A recurrence relation is a "mathematical technique" which allows us to define sequences, sets, even operations or algorithms, from particular cases to general cases. When we use this "technique", is of

fundamental importance to have attention, firstly, at the initial condition(s) – which must be known – and, on the other hand, the "recurrence equation" – which is not more than the rule that will calculate the next terms in the light of predecessors. In this talk, we introduce some special sequences of quaternions and octonions defined by a second-order recurrence, presenting some of its properties, generating function and generating matrix.

Orador: Paulo Guilherme dos Santos

Afiliação: Departamento de Matemática, Faculdade de Ciências e Tecnologia - Universidade Nova de Lisboa

Título: Acerca de um determinado método de construção de álgebras de Ockham

Resumo: Dada uma álgebra de Boole B , um número natural k e uma aplicação $\alpha : \{1, \dots, k-1\} \rightarrow \{1, \dots, k-1\}$; definimos, no reticulado B^k , uma determinada álgebra de Ockham B_{α}^k . Nesta conformidade, obtivemos uma descrição dos pontos fixos da referida álgebra de Ockham, para além de outras propriedades da referida construção.