

## Teoria de Operadores e Análise Complexa

Sessão de 90 minutos com 3 apresentações de 25 minutos cada e 5 minutos de intervalo entre cada elas:

**Título:** Relative numerical ranges

**Orador:** Janko Bracic, University of Ljubljana, Eslovénia.

**Título:** Fields of values of linear pencils and spectral inclusion regions

**Orador:** Ana Nata, Instituto Politécnico de Tomar, CMUC.

**Co-autor:** Natália Bebiano

**Título:** Langlands functoriality and K-theory for the reduced  $C^*$ -algebra of  $GL_n(\mathbb{R})$

**Orador:** Sérgio Mendes, ISCTE - Instituto Universitário de Lisboa.

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**Título:** Wiener-Hopf factorization and Q-classes

**Orador:** Cristina Câmara, IST- Universidade de Lisboa.

**Título:** On an estimate for the dimension of the kernel of a singular integral operator with non-Carleman shift and conjugation

**Orador:** Rui Marreiros, Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade do Algarve.

**Co-autor:** Ana Conceição

**Título:** An algorithm for the factorization of some classes of matrix functions

**Oradores:** Juan Carlos Sánchez Rrodríguez, Departamento de Matemática da Faculdade de Ciências e Tecnologia da Universidade do Algarve, e Paulo Alexandre Valentim Semião, Departamento de Matemática, Faculdade de Ciências e Tecnologia da Universidade do Algarve.

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**Abstracts**

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**Relative numerical ranges**

**Janko Bračič**

University of Ljubljana, Slovenia

Let  $H$  be a separable complex Hilbert space with inner product  $(\cdot|\cdot)$ . We denote by  $S_H$  the unit sphere of  $H$  and by  $B(H)$  the Banach algebra of all bounded linear operators on  $H$ . The numerical range of  $S \in B(H)$  is  $W(S) = \{(Sx|x); x \in S_H\}$ . It is well-known that  $W(S)$  is a non-empty convex subset of the disc  $\{z \in \mathbb{C}; |z| \leq \|S\|\}$ . Many important properties of an operator are encoded in its numerical range. For instance, the spectrum of  $S$  is a subset of the closure of the numerical range of  $S$ . We are interested in some parts of the numerical range of  $S$  which are specified by an operator  $T \in B(H)$ . We call them relative numerical ranges. They carry useful information about the relation between  $S$  and  $T$ . We will show that the position of zero with respect to a relative numerical range gives an information about the distance between the involved operators.

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# Wiener-Hopf factorization and Q-classes

Cristina Câmara

IST - Universidade de Lisboa

A generalization of the notion of  $\mathfrak{Q}$ -classes  $C_{Q_1 Q_2}$ , introduced in the context of Wiener-Hopf factorization, is described, and various applications to factorization problems and to the study of Toeplitz operators are presented.

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## On an estimate for the dimension of the kernel of a singular integral operator with non-Carleman shift and conjugation

Rui Marreiros

Faculdade de Ciências e Tecnologia, Universidade do Algarve

On the Hilbert space  $\tilde{L}_2(\mathbb{T})$  the singular integral operator with non-Carleman shift and conjugation  $K = P_+ + (aI + AC)P_-$  is considered, where  $P_{\pm}$  are the Cauchy projectors,  $A = \sum_{j=0}^m a_j U^j$ ,  $a, a_j$ ,  $j = \overline{1, m}$ , are continuous functions on the unit circle  $\mathbb{T}$ ,  $U$  is the shift operator and  $C$  is the operator of complex conjugation. An estimate for the dimension of the kernel of the operator  $K$  is obtained; some particular cases are considered.

This talk is based on a joint work with Ana Conceição.

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# Langlands functoriality and $K$ -theory for the reduced $C^*$ -algebra of $GL_n(\mathbb{R})$

Sérgio Mendes

ISCTE - Instituto Universitário de Lisboa

In Langlands conjectures, functoriality is a far-reaching principle, with implications in representation theory, number theory and beyond. In this talk we investigate base change and automorphic induction, two examples of functoriality, at the level of  $K$ -theory for the reduced  $C^*$ -algebra of  $GL_n(\mathbb{R})$ . As an application, the resemblance between the  $K$ -groups of  $C_r^*GL_{2n}(\mathbb{R})$  and  $C_r^*GL_n(\mathbb{C})$  will be interpreted using automorphic induction.

## References:

Langlands, R. (1989). On the classification of irreducible representations of real algebraic groups. In: P. Sally, D. Vogan (Eds.), Representation theory and harmonic analysis on semisimple Lie groups, Math. Surveys and Monographs 31, (pp. 101-170). Providence, R.I.: American Mathematical Society.

Mendes, S. On the  $K$ -theory of the reduced  $C^*$ -algebras of  $C_r^*GL_n(\mathbb{R})$  and  $GL_n(\mathbb{C})$  (preprint).

Mendes, S. and Plymen, R. Functoriality and  $K$ -theory for  $GL_n(\mathbb{R})$ , (To appear in the Münster Journal of Mathematics).

Mendes, S. and Plymen, R. (2007). Base change and  $K$ -theory for  $GL(n)$ . J. Noncommut. Geom. 1, (pp. 311–331).

# Fields of values of linear pencils and spectral inclusion regions

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Consider the linear pencil  $A - \lambda B$ , where  $A$  and  $B$  are  $n \times n$  complex matrices and  $\lambda \in \mathbb{C}$ . The field of values of a linear pencil is denoted and defined as  $W(A, B) = \{\lambda \in \mathbb{C} : x^*(A - \lambda B)x = 0, x \in \mathbb{C}^n, \|x\| = 1\}$ , where  $\|x\| = \sqrt{\langle x, x \rangle} = \sqrt{x^*x}$  is the usual Euclidean norm in  $\mathbb{C}^n$ .

In this talk we propose an efficient method for a numerical approximation of  $W(A, B)$  when one of the matrix coefficients  $A$  or  $B$  is Hermitian and  $\lambda \in \mathbb{C}$ . Furthermore, we investigate spectral inclusion regions for the pencil based on certain fields of values.

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## An algorithm for the factorization of some classes of matrix functions

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Semião

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The main purpose of this work is to present a computer algorithm for the explicit factorization, on the unit circle, of some classes of matrix functions.

This is a subject that we have been working since 2006, in which, together we supervised a master's thesis that contains a software tool for the explicit factorization of rational matrices.

It should be emphasized that the factorization of matrix functions finds applications on several fields such as, diffraction theory, differential equations, boundary value problems and operator theory, but only for a few classes of matrices the explicit method of factorization is known.